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ON THE PROBLEM OF MAGNETIC WAVE PROPAGATION AND ABSORPTION WITH REGARD FOR RELAXATION PROCESSES

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To describe the interaction of electromagnetic waves with a polarized medium, electric and magnetic relaxation times are introduced. A solution to the problem of periodic boundary regime propagation with regard for the hereditary characteristics of the absorbing medium, which qualitatively agrees with the experimental data, has been obtained.

This paper considers the electric and thermal fields created by macroscopic charges and currents in continuous media. From the practical point of view it is interesting to model local heat releases in media under the action of a high-frequency electromagnetic field. In so doing, it is necessary to take into account the fact that the energy absorption influences the electromagnetic wave propagation, since the transfer processes are interrelated.

The aim of the paper is to analyze the existing approaches to the description of the propagation of high-frequency electromagnetic waves in media with absorption and formulate a new physicomathematical model taking into account in nonstationary constitutive equations the delay between the polarization field and the external electromagnetic field.

As is known, for slow changes in the electric field the instantaneous value of current $J(t)$ is determined by the value of the electromotive force $U(t)$ at the same instant of time according to $U = RJ(t)$. At arbitrary frequencies in the oscillatory circuit the resistance $Z(\omega)$ depends on the frequency, and the function $U(\omega) = Z(\omega)J(\omega)$ is complex. Here $Z(\omega)$ is a complex resistance (or conductor impedance), to find which circuit capacity C and inductance L are introduced.

In an oscillatory circuit with continuously distributed parameters, the energy dissipation is related to the dielectric losses as a result of the frequency dependence of the relative dielectric constant $\varepsilon(\omega)$ [1]. In the general case, the value of $\varepsilon(\omega)$ is also complex, and the relation between the electric displacement and electric field vectors is of the form [1] $\mathbf{D} = \varepsilon(\omega)\mathbf{E}$, where $\varepsilon(\omega) = \varepsilon'(\omega) - i\varepsilon''(\omega)$, and ε' and ε'' are thereby determined by experiment.

At the present time, problems of dielectric heating of a continuous medium are reduced, in many cases, to the consideration of the equivalent circuit, using lumped parameters such as capacitance, inductance, loss angle [2–4], and relative dielectric loss factor [4, 5], which are found experimentally. With this approach great difficulties in establishing the temperature field of equivalent circuits arise. Polarization and the appearance of a double electric layer having a certain electric moment also take place when media with different properties contact one another. The equivalent circuits in layered media include a number of empirical "lumped" parameters: surface capacitance, surface resistance [5–7]. If sinusoidal voltage is applied to the circuit, then the flowing current will phase-shift to the voltage.

The line current can always be broken down into a dissipative component (or conduction current), which is in phase with the applied voltage, and a displacement current, which is time-shifted from the voltage. The exact physical meaning that can be attached to these two components of current largely depends on the choice of the electric equivalent circuit. There is no unique equivalent circuit — series or parallel connection of capacitor, resistor, and inductor — all is determined by a more or less adequate agreement with the experimental data.

In practice, the value of the operating capacitor capacitance C_{eff} is judged by the current. The presence of inductance in the capacitor increases its effective capacitance with increasing frequency, since in this case the current

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will increase due to the compensation of the reactance capacitive resistance by the inductive resistance. The relation between the effective capacitance and the capacitor capacitance is defined by the formula [4]

$$C_{\rm eff} = C/(1 - \omega^2 LC) .$$

This expression shows that the frequency dependence of the effective capacitance increases with increasing ω, L, and C. The presence of ohmic resistance R (e.g., resistance of the capacitor plate connected in series with the capacitor) also leads to the frequency dependence of capacitance [4]

$$C_{\rm eff} = C/(1 + \omega^2 C^2 R^2) .$$

In electrochemical capacitors, the role of one of the plates is played by a double electric layer with a specific resistance considerably exceeding the resistance of metal plates; therefore, for such capacitors a decrease in capacitance with decreasing frequency is already observed in the region of acoustic frequencies [6]. Circuits equivalent to the electrolytic capacitor are very cumbersome and have up to 12 components R, L, and C, which makes it difficult to obtain the true value of the electrolyte capacitance. In [6], experimental methods for measuring the dielectric properties of electrolyte solutions at different frequencies are given and the values of ε' and ε'' are determined, and the frequency dependences of dispersion and absorption are thereby essentially different consequences of one phenomenon — dielectric polarization inertia. Actually, the dependence ε(ω) can be explained, as mentioned above, by the presence of resistance of the double electric layer and by the fact that the electrochemical cell in the electrochemical capacitor is a system with continuously distributed parameters, in which the signal velocity is finite.

In fact, ε' and ε'' are some integral characteristics of the material at some fixed temperature and depend on the sample geometry and the thermophysical characteristics and properties of the double electric layer. As is known [1, 8], in the case of a field arbitrarily depending on temperature, any reasonable determination of absorbed energy in terms of ε(ω) turns out to be impossible. This can only be done by concretizing the time dependence of the field **E**.

For the quasi-monochromatic field we have [8]

$$\mathbf{E}(t) = [\mathbf{E}_0(t)\exp(-i\omega t) + \mathbf{E}_0^*(t)\exp(i\omega t)]/2 ,$$

$$\mathbf{H}(t) = [\mathbf{H}_0(t)\exp(-i\omega t) + \mathbf{H}_0^*\exp(i\omega t)]/2 .$$

The values of $\mathbf{E}_0(t)$ and $\mathbf{H}_0(t)$ change very slowly in time $2\pi/\omega$ and, according to [8], for absorbed energy upon averaging over the frequency ω we obtain the expression

$$\overline{\frac{\partial \mathbf{D}(t)}{\partial t}\mathbf{E}(t)} = \frac{d(\omega\varepsilon'(\omega))}{4d\omega}\frac{\partial}{\partial t}[\mathbf{E}_0(t)\mathbf{E}_0^*(t)] + \frac{\omega\varepsilon''}{2}\mathbf{E}_0(t)\mathbf{E}_0^*(t) +$$

$$+ i\frac{d(\omega\varepsilon''(\omega))}{4d\omega}\left(\frac{\partial \mathbf{E}_0(t)}{\partial t}\mathbf{E}_0^*(t) - \frac{\partial \mathbf{E}_0^*(t)}{\partial t}\mathbf{E}_0(t)\right),$$

where derivatives are taken at the "carrier" frequency ω.

Note that in the case of an arbitrary function $\mathbf{E}(t)$ its representation in the form of $\mathbf{E}(t) = \mathbf{a}(t)\cos\varphi(t)$ is difficult because it is impossible to uniquely give the amplitude $\mathbf{a}(t)$ and the phase $\varphi(t)$. It is not clear how to partition $\mathbf{E}(t)$ into cofactors a and $\cos\varphi$. If we complete the real oscillation of $\mathbf{E}(t)$ with an imaginary part $W(t)$ and go to the complex representation [9] $E'(t) = V(t) + iW(t)$, then even greater difficulties arise. In this case, $E'(t) = a(t)\exp(i\varphi t)$, where the amplitude $a(t)$, the phase $\varphi(t)$, and the instantaneous frequency $\omega = d\varphi/dt$ are defined by the known expressions

$$a(t) = \sqrt{V^2(t) + W^2(t)} ,\quad \varphi(t) = \arccos\frac{V(t)}{a(t)} = \arcsin\frac{W(t)}{a(t)} ,$$

$$\omega(t) = \frac{1}{a^2(t)}\left[V(t)\frac{dW(t)}{dt} - W(t)\frac{dV(t)}{dt}\right].$$

The time derivatives of $a(t)$ considerably complicate the operations with the passage to complex variables. The problems arising thereby are considered in more detail in [9], where it is emphasized that without unique determination of the amplitude, phase, and frequency some methods using a complex representation and pretending to a higher accuracy become incorrect.

Summarizing the foregoing, it may be stated that the calculation of dielectric losses is mainly empirical. Therefore, in modeling the electromagnetic wave propagation and absorption, one encounters fundamental difficulties. The laws of electromagnetic wave absorption in the region of the double electric layer and outside it are different. The dependences $\varepsilon(\omega)$ and $\mu(\omega)$ are not only the characteristics of the material but in many ways also functions of the process. They hold only for those experimental conditions under which they were determined. Therefore, it is difficult to describe the character of this process by the above dependences. Moreover, one should take into account that the electrophysical characteristics of the medium strongly depend on the temperature, as a rule, and, therefore, the value of dielectric losses is strongly influenced by the thermophysical properties of the material and the heat-exchange conditions. The features of the electromagnetic-wave passage through the interface with allowance for the influence of the double electric layer were considered in detail earlier in [10].

As we see it, the heat release in media under the action of nonstationary electric fields can be calculated on the basis of taking into account the interaction of the electromagnetic waves and thermal fields as a system with continuously distributed parameters based on the field equations and the energy equation.

In considering an electromagnetic field interacting with a material medium, let us make use of the Maxwell equations

$$\frac{\partial \mathbf{D}}{\partial t} + J_q = \mathrm{rot}\,\mathbf{H}, \quad \mathrm{div}\,\mathbf{D} = \rho, \quad \mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}, \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{rot}\,\mathbf{E}, \quad \mathrm{div}\,\mathbf{B} = 0, \quad \mathbf{B} = \mu_0\mathbf{H} + \mathbf{I}, \tag{2}$$

where $\mathbf{P} = (\varepsilon - 1)\varepsilon_0\mathbf{E}$ and $\mathbf{I} = (\mu - 1)\mu_0\mathbf{H}$ are the electric and magnetic polarizations, respectively.

We assume that in a continuous medium at the initial instant of time there are no space charges and they do not arise during the process. In a layered system, the space charge of the double electric layer always exists. It will be considered separately. We give the energy equation in the form

$$\rho C_p \frac{dT}{dt} = \mathrm{div}\,(k(T)\,\mathrm{grad}\,(T)) + Q. \tag{3}$$

According to [8], the electromagnetic energy converted into heat is defined by the expression

$$Q = \rho\left[\mathbf{E}\frac{d}{dt}\left(\frac{\mathbf{D}}{\rho}\right) + \mathbf{H}\frac{d}{dt}\left(\frac{\mathbf{B}}{\rho}\right)\right] + \mathbf{J}_q\mathbf{E}. \tag{4}$$

If ε, μ, and ρ are constant and $\lambda \to 0$, then the heat release is absent; therefore, the dielectric losses are associated with the introduction of $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$.

The introduction of complex variables $Z(\omega)$ and $\varepsilon(\omega)$ is a successful technique for integral processing of experimental data, though it does not permit calculating the charge of the double electric layer and local heat releases. Therefore, experimental data for the circuit provide the possibility of modeling the process only under the given experimental conditions, since the double-layer capacitance proper is a function of the process. It is the presence of capacitance that makes it impossible to characterize a continuous medium by the specific electrophysical properties alone [11]. In the case of a series connection of "small" and "large" capacitors, the resulting circuit capacitor is actually equal to the "small" one.

A characteristic feature of high frequencies is the fact that the polarization field (polarization) lags behind the change in the external field; therefore, it is expedient to determine the polarization vector from the solution of the equation $\mathbf{P}\,(t+\tau_e) = (\varepsilon-1)\varepsilon_0\mathbf{E}(t)$ with allowance for the relaxation time τ_e. Restricting ourselves to the first term of the Taylor expansion of $\mathbf{P}\,(t+\tau_e)$, from this equation we obtain

$$\mathbf{P}\,(t) + \tau_e \frac{d\mathbf{P}\,(t)}{dt} = (\varepsilon - 1)\,\varepsilon_0 \mathbf{E}\,(t)\,. \tag{5}$$

The solution of (5), provided that at the initial instant of time $\mathbf{P} = 0$, is of the form

$$\mathbf{P} = \frac{(\varepsilon-1)\,\varepsilon_0}{\tau_e}\int_{t_0}^{t} \mathbf{E}\,(\tau)\exp\left(-\,(t-\tau)/\tau_e\right)d\tau\,. \tag{6}$$

To exclude the influence of the initial conditions and transient processes, as usual, we assume that $t_0 = -\infty$. If the boundary regime lasts for a rather long time, then, due to the friction inherent in any real physical system, the influence of the initial data weakens in the course of time. Thus, we naturally arrive at a problem without initial conditions:

$$\mathbf{P} = \frac{(\varepsilon-1)\,\varepsilon_0}{\tau_e}\int_{-\infty}^{t} \mathbf{E}\,(\tau)\exp\left(-\,(t-\tau)/\tau_e\right)d\tau\,. \tag{7}$$

Consider the case of a harmonic field $\mathbf{E} = \mathbf{E}_0 \sin \omega t$. Using (7), for the electric induction vector we have

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} = \frac{(\varepsilon-1)\,\varepsilon_0}{\tau_e}\int_{-\infty}^{t} \mathbf{E}\,(\tau)\exp\left(-\,(t-\tau)/\tau_e\right)d\tau + \varepsilon_0\mathbf{E}_0 \sin \omega t =$$

$$= \frac{\mathbf{E}_0\,(\varepsilon-1)\,\varepsilon_0}{1+\omega^2\tau_e^2}\left(\sin\omega t - \omega\tau_e\cos\omega t\right) + \varepsilon_0\mathbf{E}_0\sin\omega t\,. \tag{8}$$

The electric induction vector is the sum of two physical quantities: the field strength and the polarization of a unit volume of the medium.

If the change in the substance density is small, then from formula (4) for the local instantaneous heat release we obtain, due to the action of the electric component of the field alone, the expression

$$Q = \mathbf{E}\frac{d\mathbf{D}}{dt} = \frac{\mathbf{E}_0^2\,(\varepsilon-1)\,\varepsilon_0}{1+\omega^2\tau_e^2}\left(\omega\sin\omega t\cos\omega t + \omega^2\tau_e\sin^2\omega t\right) + \lambda\mathbf{E}_0^2\sin^2\omega t\,. \tag{9}$$

But then the mean value of Q in time $2\pi/\omega$ is

$$Q = \frac{1}{2}\frac{\mathbf{E}_0^2\,(\varepsilon-1)\,\varepsilon_0}{1+\omega^2\tau_e^2}\,\omega^2\tau_e + \frac{\lambda\mathbf{E}_0^2}{2}\,. \tag{10}$$

If the relaxation time is Maxwell (in the general case it is not so) [2], i.e., $\tau_e = (\varepsilon-1)\varepsilon_0/\lambda$, then

$$Q = \frac{1}{2}\frac{\mathbf{E}_0^2\lambda\omega^2\tau_e^2}{1+\omega^2\tau_e^2} + \frac{\lambda\mathbf{E}_0^2}{2}\,. \tag{11}$$

This expression precisely coincides with the Skanavi formula [12] but was obtained from different considerations — from Eq. (5). At large frequencies $\omega \to \infty$ the heat release is no longer frequency- dependent, which agrees with (11) and experiment [12].

In using the relaxation equation for the electric field, it is also necessary to take into account the lag of the magnetic field if the magnetic polarization lags behind the change in its intensity:

$$\mathbf{I}(t) + \tau_m \frac{d\mathbf{I}(t)}{dt} = \mu\mu_0 \mathbf{H}(t) . \tag{12}$$

Consider the case where the electric and magnetic fields have the form $\mathbf{E} = \mathbf{E}(r) \exp(i\omega t)$, $\mathbf{H} = \mathbf{H}(r) \exp(i(\omega t + \varphi))$, where φ is an arbitrary phase. In a dissipative medium, changes in **E** and **H** occur with a phase shift. For the electric induction vector we have

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \int_{-\infty}^{t} \frac{(\varepsilon - 1)\,\varepsilon_0}{\tau_e} \mathbf{E}(r) \exp(i\omega t) \exp(-(t - \tau)/\tau_e)\, d\tau = \varepsilon_0 \mathbf{E} + \frac{(\varepsilon - 1)\,\varepsilon_0}{1 + i\omega\tau_e} \mathbf{E} . \tag{13}$$

From Eq. (13) the dependence $\varepsilon(\omega)$ is obtained, which, as shown above, is a consequence of the nonstationary constituent equation (5).

An analogous formula can also be obtained for the magnetic induction:

$$\mathbf{B} = \mu_0 \mathbf{H} + \frac{\mu_0 (\mu - 1)}{1 + i\omega\tau_m} \mathbf{H} . \tag{14}$$

In this case, the ordinary relations

$$\frac{d\mathbf{D}}{dt} = i\omega \mathbf{D} , \quad \frac{d^2\mathbf{D}}{dt^2} = -\omega^2 \mathbf{D} , \quad \frac{d\mathbf{B}}{dt} = i\omega \mathbf{B} . \tag{15}$$

hold.

Analysis of formulas (13), (14) shows that the values of electric and magnetic induction are determined by the whole prehistory of the change with time in the electromagnetic field, i.e., polarized media exhibit hereditary properties.

Multiply the left and right sides of Eq. (1) by $\mu_0(\mu + i\omega\tau_m)/(1 + i\omega\tau_m)$ and differentiate it with respect to time. Apply the operation rot to Eq. (2). As a result, in view of (14), (15), excluding the magnetic field from (1), (2), we obtain

$$\mu_0 \left(1 + \frac{\mu - 1}{1 + i\omega\tau_m}\right) \left[-\omega^2 \varepsilon_0 \left(1 + \frac{\varepsilon - 1}{1 + i\omega\tau_e}\right) \mathbf{E} + i\lambda\omega \mathbf{E}\right] = \Delta \mathbf{E} . \tag{16}$$

Without the above manipulations, using, as usual, in Eqs. (1) and (2) the conditions for the plane wave, it is necessary to calculate the phase shift between **E** and **H**, which in a dissipative medium with time dispersion is difficult.

In what follows, we will assume the wave to be plane and monochromatic. Consider the one-dimensional case of $E = E(x) \exp(i\omega t)$. Reducing by $\exp(i\omega t)$, from (16) we obtain the dispersion equation

$$\frac{d^2E}{dx^2} + k^2 E = 0 , \tag{17}$$

where

$$k^2 = \mu_0 \left(1 + \frac{\mu - 1}{1 + i\omega\tau_m}\right)\left[\omega^2 \varepsilon_0 \left(1 + \frac{\varepsilon - 1}{1 + i\omega\tau_e}\right) - i\lambda\omega\right] = a1 - ib1 \, . \tag{18}$$

Expressions for $a1$ and $b1$ after some manipulations can be written in the form

$$a1 = \mu_0 \varepsilon_0 \varepsilon \omega^2 \frac{(\mu + \omega^2 \tau_m^2)(\varepsilon + \omega^2 \tau_e^2)}{(1 + \omega^2 \tau_m^2)(1 + \omega^2 \tau_e^2)} - \frac{\mu_0 \omega \tau_m (\mu - 1)}{1 + \omega^2 \tau_m^2} \left[\frac{\varepsilon_0 \omega^3 \tau_e (\varepsilon - 1)}{1 + \omega^2 \tau_e^2} + \lambda\omega\right],$$

$$b1 = \mu_0 \frac{(\mu + \omega^2 \tau_m^2)}{(1 + \omega^2 \tau_m^2)} \left[\frac{\varepsilon_0 \omega \tau_e (\varepsilon - 1)\, \omega^2}{1 + \omega^2 \tau_e^2} + \lambda\omega\right] + \mu_0 \varepsilon_0 \frac{(\varepsilon \omega^2 \tau_e)}{(1 + \omega^2 \tau_e^2)^2} \frac{\omega^3 \tau_m (\mu - 1)}{(1 + \omega^2 \tau_m^2)} \, . \tag{19}$$

According to [13], to take into account the lag, it is enough to determine the dependence $\varepsilon(\omega)$. In so doing, the equation of telegraphy for the electric field vector is assumed to be valid [14]. For the magnetic field in the constituent equation the lag is neglected [13]. It can easily be shown that in this case the eigenvalue problem will have a different form.

Consider the problem on the periodic boundary regime propagation on the interval $(0, l)$ analogously to [15] but taking into account the relaxation

$$E(0, t) = 0 \, , \quad E(l, t) = A \exp(i\omega t) \, . \tag{20}$$

Assuming $E(x, t) = E(x) \exp(i\omega t)$, for Eq. (17) we have

$$E(0) = 0 \, , \quad E(l) = A \, . \tag{21}$$

Solving (17) with the boundary conditions of (20), (21), we find $E(x) = C1 \sin kx$. At $x = l$ $C1 = A \sin kl$, so that [15]

$$E(x) = A \frac{\sin kx}{\sin kl} = X1(x) + iX2(x) \, , \tag{22}$$

where $X1$ and $X2$ are the real and imaginary parts of $E(x)$. The sought solution can be given in the form

$$E(x, t) = [X1(x) + iX2(x)] \exp(i\omega t) = U1(x, t) + iU2(x, t) \, ,$$

where $U1(x, t) = X1(x) \cos \omega t - X2(x) \sin \omega t$; $U2(x, t) = X1(x) \sin \omega t + X2(x) \cos \omega t$. The solution of the general problem without the initial conditions at $E(0, t) = \mu_1(t)$ and $E(l, t) = \mu_2(t)$ is defined as the sum of two terms for each of which one of the boundary conditions is inhomogeneous [12].

For problem (20), (21), where on the sample surface with $x = l$ a periodic electric field is active, $E(l, t) = A \exp(i\omega t)$ and on its opposite side with $x = 0$, $E(0, t) = 0$, the values of the function $|E(x)|/A = |\sin kx \sin kl|$ were obtained for various values of k, thickness l, conduction λ, and relative permittivity ε.

In this case, the equality to zero of the electric field strength corresponds to the zero charge flow. Indeed, the charge flow is defined by the sum of the conduction current and the displacement current. If $E(x, t) = E(x) \exp(i\omega t)$, then at $x = 0$ $E(0) = 0$ should take place. This condition is strictly fulfilled for a semibounded dissipative medium. Therefore, in the calculations the sample thickness was assumed to be fairly large ($l = 1$ m), where the influence of the geometry on the $E(x)$ distribution is negligible.

Figure 1 gives the dependence of the electric field dimensionless amplitude on the sample thickness $|E(x)|/A$ for various values of the specific conduction (a) and magnetic permittivity (b). The given data show that an increase in λ and μ leads to a larger absorption of the electric wave, which agrees with the experimental data. Indeed, according to the experimental data of [16], the electric field in the microwave region decays exponentially. At the same time, when the classical solution without introducing the relaxation time is used [15] and for $\lambda = 0$ the solution is linear (the diagram of the linear function is not given here).

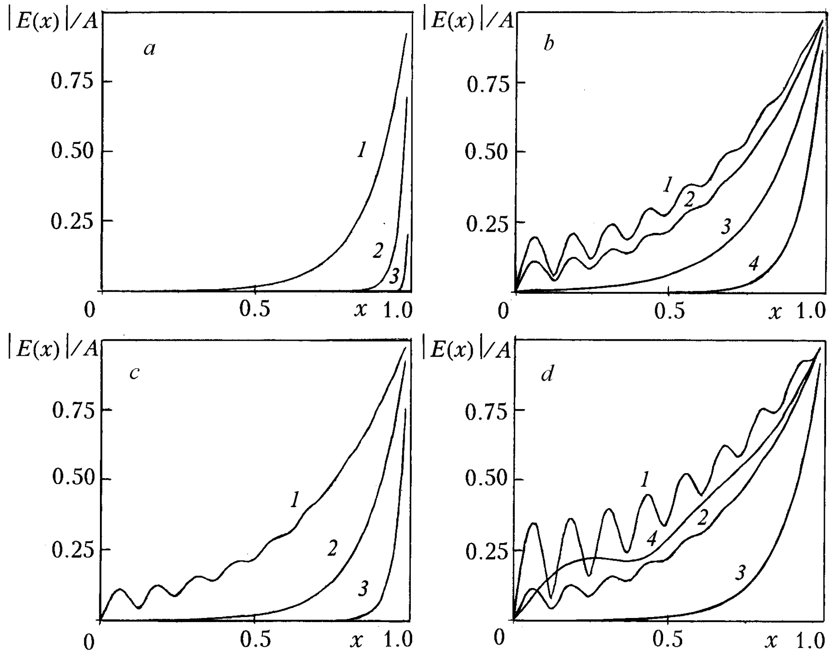

Fig. 1. Sample-thickness distribution of the electric field $E(x)$ dimensionless amplitude ($f = 2 \cdot 10^9$ Hz, $l = 1$ m, $\varepsilon = 15$) for various values of electrical conduction — a [1) $\lambda = 0$; 2) 1; 3) 20 $\Omega \cdot m^{-1}$], permeability — b [1) $\mu = 1$; 2) 10; 3) 50; 4) 200], and relaxation times — c, d [1) $\tau_m = 10^{-7}$; 2) 10^{-8}; 3) 10^{-9} sec (c); 1) $\tau_e = 0.5 \cdot 10^{-10}$; 2) 10^{-8}; 3) 10^{-9}; 4) 10^{-8} sec (d)]. $\mu = 5$, $\tau_e = 10^{-9}$; $\tau_m = 10^{-7}$ sec (a); $\lambda = 0$; $\tau_e = 10^{-10}$; $\tau_m = 10^{-7}$ sec (b); $\mu = 0$; $\lambda = 0$; $\tau_e = 10^{-10}$ (c), and $\tau_m = 10^{-7}$ sec (d).

Figure 1c, d shows the dependences of the dimensionless amplitude of the electric field $|E(x)|/A$ on the sample thickness for various values of the relaxation time of the electric and magnetic fields. Note that the magnetic relaxation time is, as a rule, an order of magnitude smaller than the electric relaxation time. Analysis of the curves given in Fig. 1c shows that a decrease in the magnetic relaxation time leads to a large signal attenuation. Along with this, the dependences given in Fig. 1d show that a resonance time of electric relaxation, at which the wave absorption is maximum, exists. If the electric relaxation time is fairly small, then in the medium effects characteristic of the standing wave with the presence of maxima and minima of the electromagnetic wave amplitude can arise.

Note that in a dissipative medium there is always a phase shift between **E** and **H**. The proposed computing method based on reduction of the field equations to one equation of telegraphy for the electric field vector does not require knowledge of the value of this shift. In the limit at $\omega \to \infty$ the functions $\varepsilon(\omega)$ and $\mu(\omega)$, according to formulas (13) and (14), tend to unity due to the fact that as a sufficiently rapid change in the field the polarization processes have no time at all to proceed. At the same time, the dispersion equation (18) for k^2 as $\omega \to \infty$ gives the dependence of the values of $a1$ and $b1$ on the field frequency. This paradox is due to the fact that in finding the real and imaginary parts of k^2, we multiply and divide the denominators by the complex-conjugate quantities $(1 - i\omega\tau_e)^{-1}$ and $(1 - i\omega\tau_m)^{-1}$, which as $\omega \to \infty$ are indeterminate. However, it should be remembered that the upper boundary of the frequency should satisfy the condition $\omega << c/\delta$.

In conclusion, note that Eq. (5) is based on the classical Debye model [18] pertaining to substances whose particles have a permanent electric dipole moment. The above mechanism of polarization consists of partial alignment of dipoles along the electric field, which is opposed by the process of dipole disorientation because of thermal collisions. The restoring "force," according to Eq. (5), does not lead to electric polarization oscillations. It acts as if the permanent electric dipoles were characterized by a strong attenuation. The relaxation Debye polarizability is characteristic of molecules of many liquids and solids [18]. Initially, the polarization aggregates of Debye oscillators return to the equilibrium state according to the expression

$$\mathbf{P}(t) = \mathbf{P}(0) \exp(-t/\tau_e) .$$

Unlike a homogeneous medium, in which the volume charge can be ignored, at the contact boundary of media with different electrophysical properties a spatial distribution of electric charges — a double layer whose effective thickness has molecular dimensions — is established. In [10, 17, 19], it is shown that in this case at the interface equality of total currents and energy fluxes with allowance for thermoelectric phenomena should be realized. The expressions for the charge and energy fluxes will be of the form

$$\mathbf{J}_d = \lambda(T)\left(\alpha(T)\frac{dT}{dx} + \mathbf{E}\right) + \frac{\partial \mathbf{D}}{\partial t} , \quad \mathbf{J}_h = k(T)\frac{dT}{dx} + \mathbf{J}_q Ï . \tag{23}$$

Using (23) and (1)–(3), one can obtain an interrelated system of equations describing the interaction of thermal and electric phenomena in the region of the double electric layer. This approach permits calculating the charge of the double electric layer [10, 17, 19].

At the present time, for a monochromatic wave propagating in a homogeneous medium with a small attenuation coefficient, an expression of the form $Q = \omega\mathbf{E}_0^2/2$ is often used [16]. In so doing, it is assumed that the wave length is not only small compared to the attenuation length Δ (e-fold decrease in the amplitude), but the stronger condition $\Delta >> l$, where l is the linear size of the medium volume under consideration in the direction of the wave propagation should also be met [20]. Moreover, plane monochromatic waves, according to [20], are considered away from the media interface.

The proposed computing method does not contain the above limitations and can be used to model waves with a strong attenuation, including the region close to the media interface.

Thus, to model the propagation and absorption of electromagnetic waves in a polarized medium and calculate the dielectric losses, it is suggested to use the telegraph equation for the electric field vector and the heat-conduction equation with allowance for the thermoelectric phenomena. The polarization current is thereby assumed to be a component of the conduction current.

To model thermal fields in a continuous medium, it suffices to give the charge and energy fluxes at the interfaces as well as information about its specific thermophysical and electrophysical properties. The introduction of such parameters as capacitance, surface capacitance, surface resistance, loss angle, real and imaginary parts of the relative dielectric constant, which give only some averaged integral characteristics of and a continuous medium, is not needed.

In this work, for the first time an attempt has been made to take into account the lag of the polarization field and the external electromagnetic field that is due to not only the electric, but also the magnetic phenomena of dipole relaxation. We have obtained a solution to the problem on the periodic boundary regime propagation with regard for the hereditary characteristics of the polarized medium, which qualitatively agrees with the experimental data.

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NOTATION

B, magnetic induction vector; **D**, electric displacement vector; **E**, electric field vector; **H**, magnetic field vector; $\mathbf{J}_q$, charge flux density; **I**, magnetic polarization vector; **P**, electric polarization vector; A, maximum value of electric field amplitude; a, amplitude; C, circuit capacitance; C_p, heat capacity; $\mathbf{E}_0$, electric field amplitude; $\mathbf{H}_0$, magnetic field amplitude; J, instantaneous value of current intensity; $k(T)$, heat-conductivity coefficient; l, linear size of a region; L, circuit inductance; Q, electromagnetic energy dissipation; R, circuit resistance to direct current; Pr, Peltier coefficient; t, time; T, temperature; U, electromotive force; x, coordinate; Z, complex resistance (conductor impedance); α, specific thermoelectromotive force; ε, relative permittivity; ε' and ε'', real and imaginary parts of relative permittivity; ε_0, electric constant; δ, characteristic atomic size; λ, specific electric conduction; ρ, density; τ_e and τ_m, electric and magnetic relaxation time; φ, phase; μ, relative permeability; μ_0, magnetic constant; ω, cyclic wave frequency. Subscripts: 1, first medium; 2, second medium; 0, initial state; eff, effective; e, electric; m. magnetic; d, displacement; q, charge; h, heat; *, complex conjugate quantity; p, pressure.

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